STRESS CONCENTRATION IN FIBROUS COMPOSITE MATERIAL

Carlos Roberto Santos Alves

NAVAL POSTGRADUATE SCHOOL

Monterey, California

THESIS

STRESS CONCENTRATION
IN
FIBROUS COMPOSITE MATERIAL

by

Carlos Roberto Santos Alves

June 1975

Thesis Advisor: M. H. Bank

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(20. ABSTRACT Continued)

nondimensional parameters, one representing the eccentricity of the holes, and the other representing the influence of the size of the holes. The graphs plotted for K_{tg} permit their use for thin plates of different dimensions.

Stress Concentration
in
Fibrous Composite Material

by

Carlos Roberto Santos Alves
Commander, Brazilian Navy
B.S., Naval Postgraduate School, 1975

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ABSTRACT

This thesis work involves an initial study on Stress Concentration caused by holes out of center in thin plates of finite width made of Unidirectional and Crossply Fiberglass Composites.

The results are reported under the form of the Stress Concentration factor K_{tg} as a function of two nondimensional parameters, one representing the eccentricity of the holes, and the other representing the influence of the size of the holes. The graphs plotted for K_{tg} permit their use for thin plates of different dimensions.

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A	Cross section area
a	Hole diameter
b	Width of the plate or specimen
c	Distance from the center of the hole to the near edge of the plate
d_1	Distance from the center of the outer gage to the edge of the hole
d_2	Distance from the center of the inner gage to the edge of the hole
E	Young's Modulus
e	Distance from the center of the hole to the far edge of the plate (eccentricity)
G._	Gage abbreviation followed by its number. If the letter H appears in sequence it means that the gage is installed beside the hole
K_{tg}	Stress concentration factor for uniaxial tension (SH/S)
$[K_{tg}]_E$	Experimentally determined stress concentration factor
$[K_{tg}]_T$	Approximate theoretical stress concentration factor, determined by the extrapolation curve
P	Load applied by the tension test machine
r	Distance from center of the hole along transverse line
$(r/a)_G$	Value of r/a, at the location of the strain gage
S	Average stress applied by the grips ($S = P/(b\times t)$)
S_H	Stress at the edge of the hole
t	Thickness of the plate or specimen
ε	Strain
ε_H	Strain at the edge of the hole

ϕ Function

ω Uncertainty value associated with a particular measurement

ABBREVIATIONS

in Inch

lbf Pounds-force

psi Pounds-force per square inch

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This thesis was developed and conducted under a cooperative agreement between the Mechanical and the Aeronautics Department, in the U.S. Naval Postgraduate School, Monterey, California.

The material used was fabricated in the Aeronautics Department Composites Shop under the supervision of Professor Milton H. Bank.

The preparation of specimens, consisting of machining operations and installation of gages, was done in the Mechanical Engineering Department shop. The research work was performed in the Mechanical Engineering Department Laboratory, located in Halligan Building.

It has been evident, during the period of approximately three quarters since the beginning of this research, as a constant, the cooperative help received from Mr. Kenneth Mothersell and technicians of the Mechanical Engineering Department Shop.

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The author also wishes to express his appreciation to Professor David Salinas, Representative of Mechanical Engineering Department before this thesis and Professor Milton H. Bank, Thesis Advisor, for all encouragement and assistance rendered throughout this student research on composite materials.

This thesis is dedicated to my wife Sylvia.

I. INTRODUCTION

Composite materials have been in use for a long time. Materials consisting of two distinct components or phases combined in adequate proportion to achieve properties more convenient to suit man's needs, and presenting a higher strength than one of its phases alone, have been known since the earliest times of human history. One example of such type of material appears in ancient history when the Egyptians glued laminae of wood (a natural composite) into laminates as early as 1500 BC. Another example in ancient history is the mongol bow (about 800 BC) that was made of a composite of animals' tendons, silk and wood, bonded together by means of an adhesive [1]. The Israelites trod straw into mud, and from such composite they made bricks for the Pharaoh [2]. In the area of arms manufacture, after about 1500 AD, manufacturers around the world made use of iron and steel composite laminates [3].

In recent times, composites have gained a special place among the most important of the modern construction materials, and the growing needs in new engineering applications seem to create a favorable environment for the development of such materials.

In the modern development of composites, new fibers like glass, boron, borsic, graphite, Kevlar and others with high strength and/or stiffness-to-weight ratios, have placed

composite materials in a competitive position with respect to other materials, for almost any construction assignment.

According to Paul F. Bruins [6], "A materials revolution is in progress which may have as much impact on the course of civilization as the materials revolution that occurred in the change from the Bronze to the Iron Age."

Information on the behavior of metallic structures has been obtained systematically since 1830 [4], and today both designer and practicing engineers are confident of having available accurate data in this field.

Unfortunately, however, for the engineer the data base on composite materials is still in its infancy, and this imposes some constraints upon their application.

Also engineers understand the limits of applicability of isotropic materials, while the same does not occur in relation to composite materials.

Project Composites Recast [5] stated that "Lack of confidence is recognized as a primary inhibiting factor in the widespread use of advanced composites... Confidence in the performance and reliability of composite structures is, of course the primary requirement."

This lack of information constituted a motivation for the study in this thesis. In an effort to expand the data base on composite materials, it was decided to study the problem of stress concentration around holes out of center in thin plates of finite width.

II. BACKGROUND

It is well known that whenever a hole is made in a plate the stress field is distorted in such a way that a stress concentration occurs as a result. In many cases in engineering constructions it is necessary to cut such holes, for example, to apply fasteners or to create a passage way, and such cuts will ever be present, especially in aeronautic and naval constructions.

The classical problem of a circular hole in a plate of finite width, made of isotropic material, under axial tension was solved by G. Kirsch in 1898 [7]. The case of a circular hole near the edge of a semi-infinite plate under uniaxial tension parallel to the edge was analyzed by G. B. Jeffrey [8]. Many others have solved similar problems, and compilations of their work are found in the books by R. E. Peterson [9] and G. N. Savin [10].

The problem of stress concentrations around a circular hole out of center in a plate of finite width, made of isotropic material, has been solved by S. Sjostrom [14]. The similar problem of a plate made of composite material is more difficult, and a closed form solution for it is not available to the engineer. This problem has been treated by two approaches: first, using the theory of linear anisotropic elasticity as discussed by Lekhnitskii [11] and Savin [10]; and second, using the finite element method

(e.g. 12). In particular, no results are available for the effect of proximity of the hole to the edge. Thus, it was decided to investigate the problem experimentally. It was also expected that an economical program of tests in this area could contribute some useful data and qualitative results to a follow-up research program.

III. SCOPE OF THE TEST PROGRAM

The objective of this thesis work was to take data which would permit drawing a diagram of the stress concentration factor K_{tg} at the edge of the hole at a position 90° from the longitudinal axis as a function of two nondimensional parameters involving, respectively, the influence of the hole size, and the influence of the eccentricity of the hole, similar to the diagram of Figure 88 in Ref. 9.

It was decided to restrict the experiments to only two types of lay-ups, unidirectional and cross-ply (0°/90°), since time and money were limited. Still as an economic measure and also to minimize the time involved in specimen manufacture it was decided to re-use the same plate of laminate for subsequent experiments, enlarging the holes corresponding to a specific eccentricity after each test. Subsequently, the eccentricity parameters were varied, giving a total of nine experiments per type of laminate. This procedure presented as a by-product the advantage of minimizing variations in results caused by material inconsistencies, since all specimens representing a particular type of laminate were cut from the same basic laminate.

It was also decided that a total of eighteen specimens (nine unidirectional, and nine cross-ply) would be a reasonable number to provide a minimum amount of data necessary to permit the plotting of the desired curves.

All the data to be obtained was intended to be valid for short term loading (creep was not taken into account) and for relatively low loading rates.

Since the specimens were made of very thin plates, the strain data was taken by means of bonded resistance strain gages of foil type glued to the surface of the plate, as close as possible to the edge of the hole.

IV. DESCRIPTION OF THE SPECIMENS

The specimens used in this testing program had nominal dimensions of 15 inches by 5 inches, and were made of epoxy glass reinforced plastic laminates, with one circular hole displaced from the center, as shown in Fig. 1.

The representative group of specimens had a programmed distribution of parameters, as shown in Table 1.

Unidirectional specimens were 0.025 inches thick, having three plies, all oriented along the tensile axis of the specimen. The cross-ply specimens were 0.045 inches thick, having a total of five plies, two in the tension direction and three at 90° to it.

One and one-half inch wide end tabs, made of glass-cloth and epoxy (reinforced plastic) surfaced with thin copper plate were glued to each end of the specimens, as indicated in Fig. 1.

These tabs were used to spread the load as uniformly as possible over the specimen ends, preventing the occurrence of cracks caused by tightening of the grips.

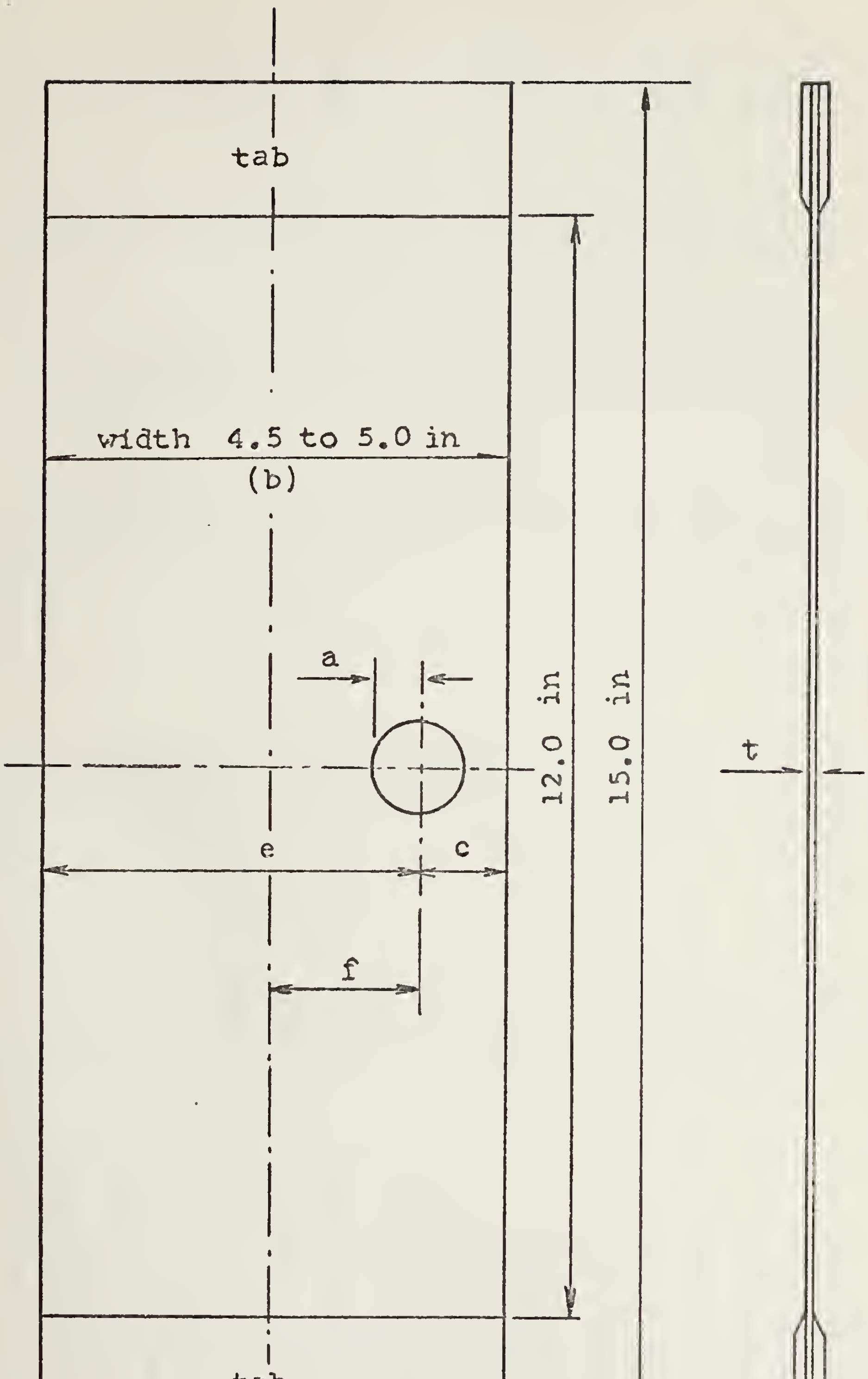

FIGURE 1. BASIC STRESS CONCENTRATION SPECIMEN DIMENSIONS

TABLE 1. SPECIMENS DIMENSIONS

Specimen Number	Type	Hole Radius (a) Inches	Hole Center Location (f) Inches	Hole Location Parameters [Ref. 9]			
				c	e	a/c	e/c
IAI	Unidirectional	0.25	1.50	0.85	3.85	0.29	4.53
IAII	Unidirectional	0.25	1.00	1.35	3.35	0.19	2.48
IAIII	Unidirectional	0.25	0.50	1.85	2.85	0.14	1.54
IBI	Unidirectional	0.188	1.50	0.85	3.85	0.22	4.53
IBII	Unidirectional	0.188	1.00	1.35	3.35	0.14	2.48
IBIII	Unidirectional	0.188	0.50	1.85	2.85	0.10	1.54
ICI	Unidirectional	0.125	1.50	0.85	3.85	0.15	4.53
ICII	Unidirectional	0.125	1.00	1.35	3.35	0.09	2.48
ICIII	Unidirectional	0.125	1.50	1.85	2.85	0.05	1.54
IIAI	Cross-ply	0.25	1.50	0.94	3.94	0.27	4.19
IIAII	Cross-ply	0.25	1.00	1.44	3.44	0.17	2.39
IIAIII	Cross-ply	0.25	0.50	1.94	2.94	0.13	1.52
IIBI	Cross-ply	0.188	1.50	0.94	3.94	0.20	4.19
IIBII	Cross-ply	0.188	1.00	1.44	3.44	0.13	2.39
IIBIII	Cross-ply	0.188	0.50	1.94	2.94	0.10	1.52
IICI	Cross-ply	0.125	1.50	0.94	3.94	0.13	4.19
IICII	Cross-ply	0.125	1.00	1.44	3.44	0.09	2.39
IICIII	Cross-ply	0.125	0.50	1.94	2.94	0.06	1.52

V. MANUFACTURE OF SPECIMENS

The specimens used in this test program were cut from glass-epoxy composite plates, manufactured in the Naval Postgraduate School Aeronautics Department Composites Laboratory. A complete description of this Laboratory and of the procedure followed in laying-up and curing composite plates for test is given in Ref. 13.

The fabrication of the laminates consisted of a curing cycle under pressure of 50 psi. The temperature was increased from the ambient temperature of 300 °F at a rate of 5.37 °F/minute, maintained at that temperature for forty minutes, and then allowed to cool under pressure to the ambient temperature.

Each cured laminate had dimensions of about sixteen inches by sixteen inches, and from each laminate three specimens were cut with a nominal width of five inches each, and a length of fifteen inches. This procedure was followed for both unidirectional and cross-ply specimens.

Each one of these specimens represented a series corresponding to three different eccentricities, e, as shown in Table 1.

Each of the specimens was prepared with grip tabs made of glass fabric-epoxy laminate surfaced with thin copper plate, as shown in Fig. 1. The grip tabs were glued to both sides of the specimen ends and no screws were used.

The holes were cut and after each test enlarged by means of a drill for holes up to 1/2 inch, and by means of an end mill for holes of 5/8 inch.

The machining operations were made with the specimens firmly pressed between two plates made of wood.

After the laminates were cured their surfaces did not receive any special treatment besides the removing the scales. Sand paper was not used to improve the final finishing, except over the areas where the strain gages were glued on.

All gages were installed by the Mechanical Engineering Laboratory Shop and the instructions recommended by the manufacturers were followed. All foil gages were glued with Eastman 910 glue.

The gages used for the purpose of providing information relative to the stress concentration factors were of the foil type, while the ones destined to provide only qualitative information were of the paper type. This measure was taken in order the minimize the cost.

The strain gages were located as shown in the general examples of Figs. 2 and 3. The details of gage installation for each one of the specimens are described in Appendix D, where distance d, from the center line of the gage to the edge of the hole, is reported.

Most specimens were equipped with two gages, one for each side of the hole as shown in Figure 2, while some others were more fully equipped, having a row of gages along the tranverse

line containing the center of the hole, and along a tranverse line, half way between the hole and the grip tabs, as shown in Fig. 3.

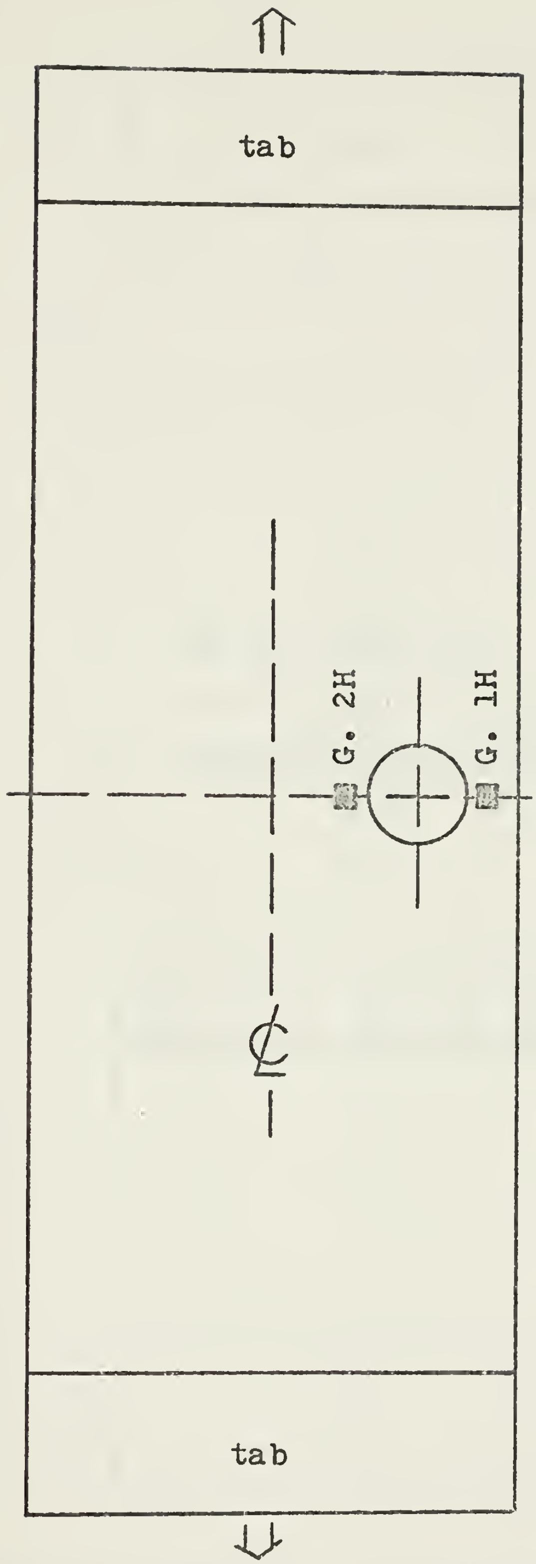

FIGURE 2. TYPICAL GAGE ARRANGEMENT

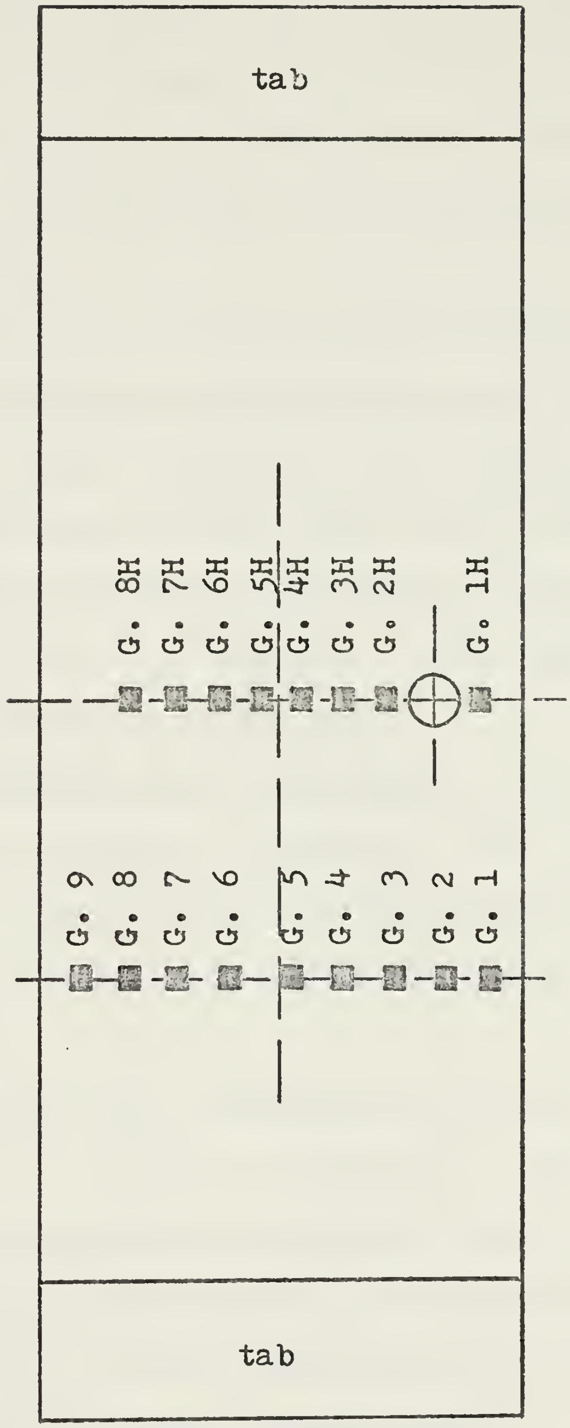

FIGURE 3. GAGE ARRANGEMENT FOR TRANSVERSE STRESS DISTRIBUTION

VI. INSTRUMENTATION AND TESTING APPARATUS

The Tension Test Machine used was the Baldwin Southwark Division with a maximum capacity of 60,000 pounds, located in the Mechanical Engineering Laboratory. The machine was operated in the low range, with the loading process controlled manually.

The gage indicator was of the Baldwin Type N, belonging to the Mechanical Engineering Laboratory, and was installed together with a Baldwin Switching and Balancing Unit.

The gages used were of the types listed below:

Type FAE-12S131-SRY with resistance of 120 ± 0.2 ohms, and gage factor of 2.07 ± 1 percent. This type of gage was used with the verification specimen.

Type C12-1x1-M30D with a resistance of 120 ± 0.5 ohms and a gage factor of 2.02 ± 1 percent. This type of gage was used with the verification specimen and also with the main group of specimens, only during the initial period of the test program.

Type EA-06-062AP with a resistance of 120 ± 0.15 ohms and a gage factor of 2.085 ± 0.5 percent. This type of gage was used with the specimens of the main group.

The gages were manufactured by MicroMeasurements, Romulus, Michigan, except for the C12 gages which were SR-4 gages made by BLH Electronics, Waltham, Massachusetts.

VII. EXPERIMENTAL PROCEDURE

Prior to beginning the main test series a verification specimen was fabricated and tested. This specimen was numbered AIAI, and gave information about the test procedures and about the existence of uniformity of the stress field across a transverse line between the hole and the grip tabs.

From the information given by the verification specimen, the first hole size to be tested in the main program was 1/4 inch, in order to minimize the disturbance caused in the stress field by the size of the hole.

The specimens were installed in the tension test machine, using grips of the "press and pull" type with two jaws, made of steel, that covered about 90 percent of the tab areas. The arrangement of the jaws permitted good alignment of the specimens along the vertical line.

During the experiments the load was raised to values between 500 and 600 pounds as many times as necessary to obtain approximately no zero shift, with a tolerance of ± 20 microstrains. This zero shift was taken into account in the error analysis.

Also during the tests, load levels in increments of 50 pounds from zero to 500 pounds were applied in random order.

During the experiments, it was found necessary to fabricate two extra specimens, one for unidirectional and another for

cross-ply laminates. These specimens were tested in order to obtain curves of stress decay along the transverse line containing the center of the hole. These curves were supposed to provide information for extrapolation of strains, from the gage positions to the edge of the holes. These specimens were identified as IDIII and IIDIII, respectively, for unidirectional and cross-ply types.

During the final period of experiments, another specimen, made of aluminum with the number ALUM-1 was fabricated. This specimen had two gages, one installed inside the hole and another installed on the surface of the plate close to the edge of the hole. The purpose of testing the aluminum specimen was to obtain a calibration and a verification of the overall testing procedure.

The experiments are valid for low loading rates and low stresses as well. The low values of the loads applied were limited by the maximum deformation permitted for the gages, and also by the fact that high values of the loading can cause a minor stress concentration in the matrix that eventually produces cracks.

The strain gage circuit used was the simple bridge arrangement with temperature compensation. Beside the natural temperature compensation of the system, a shielding skirt was used to insulate the specimen from environmental convection currents.

As a last, but very important part of the experimental procedure it was noted that substantial creep did not occur.

The verification specimen was tested for about three hours under constant loading and the strain readings did not show any creep.

VIII. PRESENTATION OF RESULTS

A. DATA ACQUIRED

The data obtained in this test program may be found in Appendix D. The stress concentration factors computed from this data and extrapolated to the edge of the hole by the method described in Part B of this section are presented in Figs. 4 and 5, respectively for unidirectional and cross-ply groups of specimens. These graphs present the stress concentration factor K_{tg} as a function of the nondimensional parameters, a/c, and e/c. Parameter a/c represents the influence of the size of the hole, and e/c represents the influence of the eccentricity of the hole. The continuous line in each graph shows the variation of K_{tg} as a function of the nondimensional a/c for the case of isotropic material and for e/c = 2.5. The dashed lines represent an expectation for each one of the composites, but do not rely on any theoretical foundation.

B. EXTRAPOLATION OF THE DATA

Due to the fact that the plates were very thin, it was not possible to install the gages inside the holes. Thus it was necessary to devise a procedure for extrapolating the data obtained from gages placed at the side of the holes.

In view of this problem, it was felt necessary to devise an extrapolation method in order to obtain a value of the stress concentration factor K_{tg} at the edge of the hole,

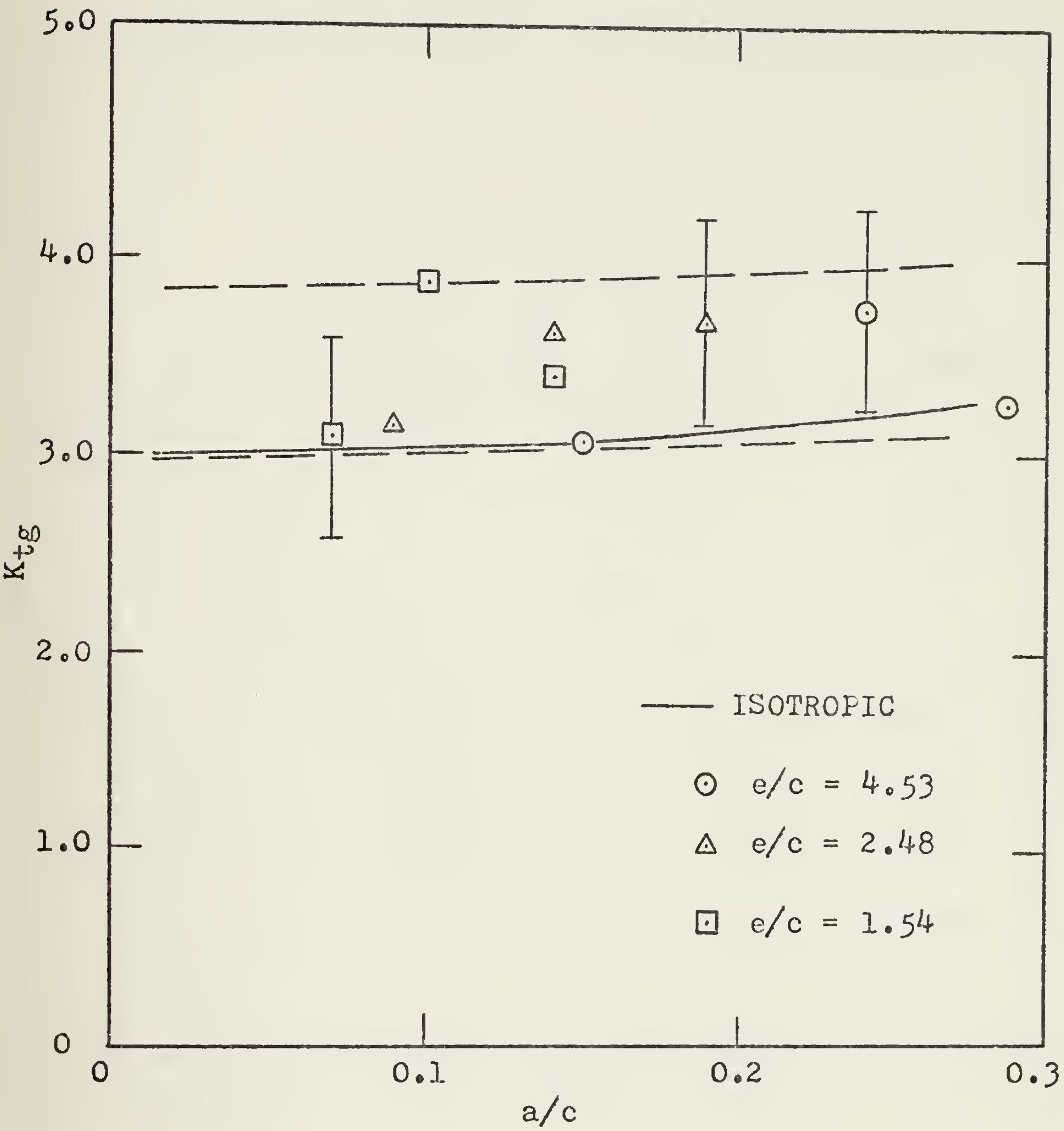

FIGURE 4. AVERAGE STRESS CONCENTRATION FACTOR K_{tg} FOR TENSION CASE OF UNIDIRECTIONAL FIBERGLASS. FLAT BAR WITH CIRCULAR HOLE DISPLACED FROM THE CENTER

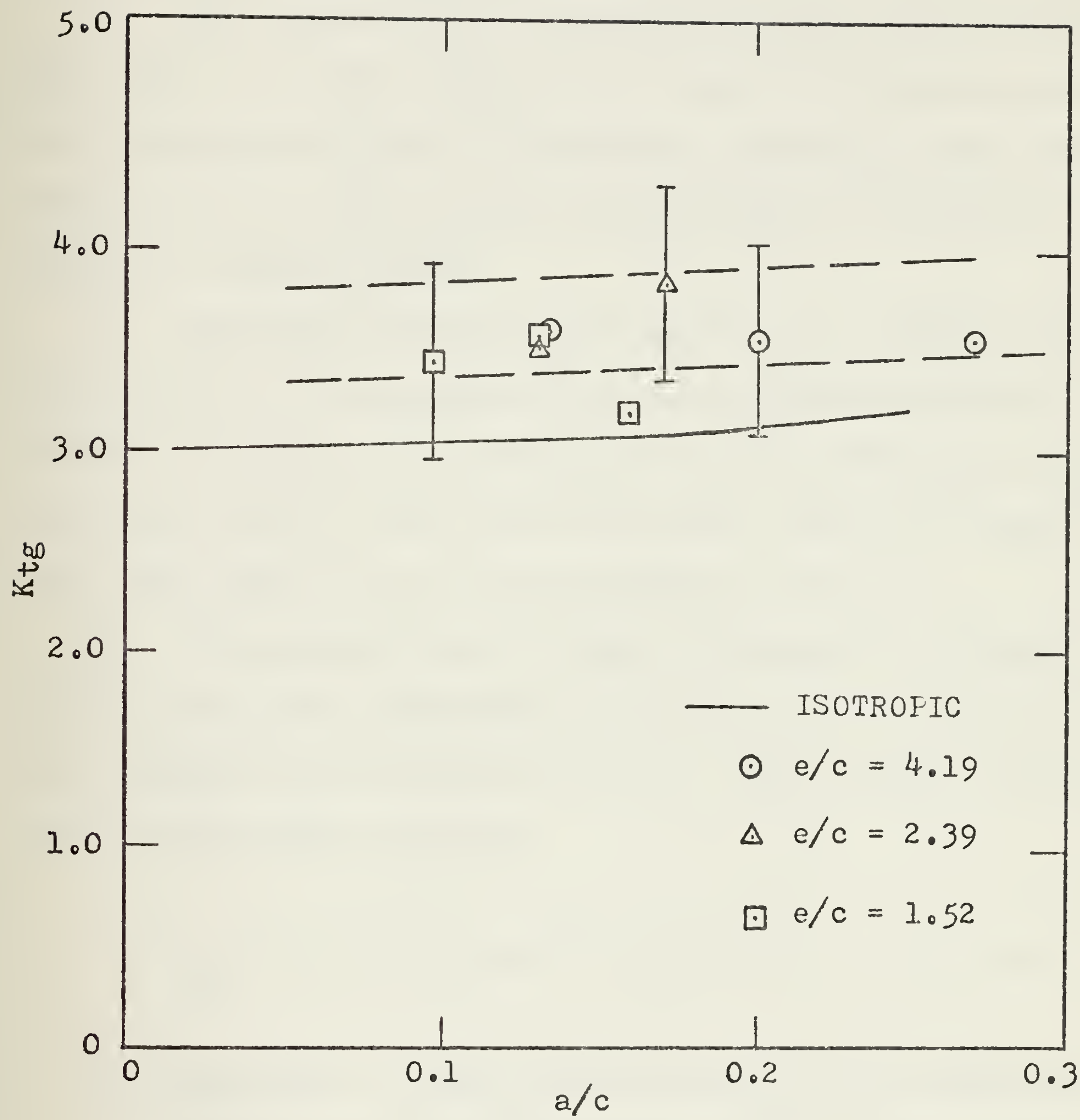

FIGURE 5. AVERAGE STRESS CONCENTRATION FACTOR K_{tg} FOR TENSION CASE OF CROSSPLY FIBERGLASS. FLAT BAR WITH CIRCULAR HOLE DISPLACED FROM THE CENTER

based on the data obtained from the gages located beside the hole. This method of extrapolation is described below.

The first part of the method consisted of finding an extrapolation curve, as follows:

1. The values of K_{tg} at the edge of the hole were taken from Ref. 17. These values are 4.2 and 3.5 respectively, for unidirectional and for cross-ply fiberglass, and were obtained from a theoretical solution based on Rheology, for plates of infinite width.

2. The Kirsch solution [20] gives a K_{tg} value of 1.02 at $r/a = 5$. It was assumed that this distance from the center of the hole was sufficient for the differences between the Kirsch solution and the anisotropic case to be small. Thus, the value of 1.02 was assigned at $r/a = 5$.

3. The general form of Kirsch solution was assumed to be valid for both composites, unidirectional and cross-ply, with unknown coefficients A_i and B_i. Thus, Kirsch solution was written under the form:

$$K_{tg} = 1.02 + A_i(a/r)^2 + B_i(a/r)^4 \qquad (i = 1,2)$$

The subscript i, refers to each one of the composites.

4. Based on the two conditions given in items 1 and 2 above, the coefficients A_1 and B_1 for unidirectional, and A_2 and B_2 for the cross-ply laminates were found.

The resultant equations are:

$$K_{tg} = 1.02 + 0.4188(a/r)^2 + 2.0312(a/r)^4 \quad \text{for cross-ply}$$

$$K_{tg} = 1.02 + 0.3875(a/r)^2 + 2.8125(a/r)^4 \quad \text{for unidirectional}$$

These curves are shown in Fig. 6.

The second part of the extrapolation method involved the determination of K_{tg} at the hole, based on the extrapolation curve just obtained and on the experimentally determined K_{tg} at the gage position. To accomplish this the following steps were taken (see Fig. 7):

1. A secant was drawn from point A ($[K_{tg}]_T$ at $(r/a)_G$) to point B ($[K_{tg}]_T$ at $r/a = 1$).

2. A line parallel to AB was drawn through point C ($[K_{tg}]_E$ at $(r/a)_G$). The intersection of this line with the ordinate gives the extrapolated value of K_{tg}, which was sought.

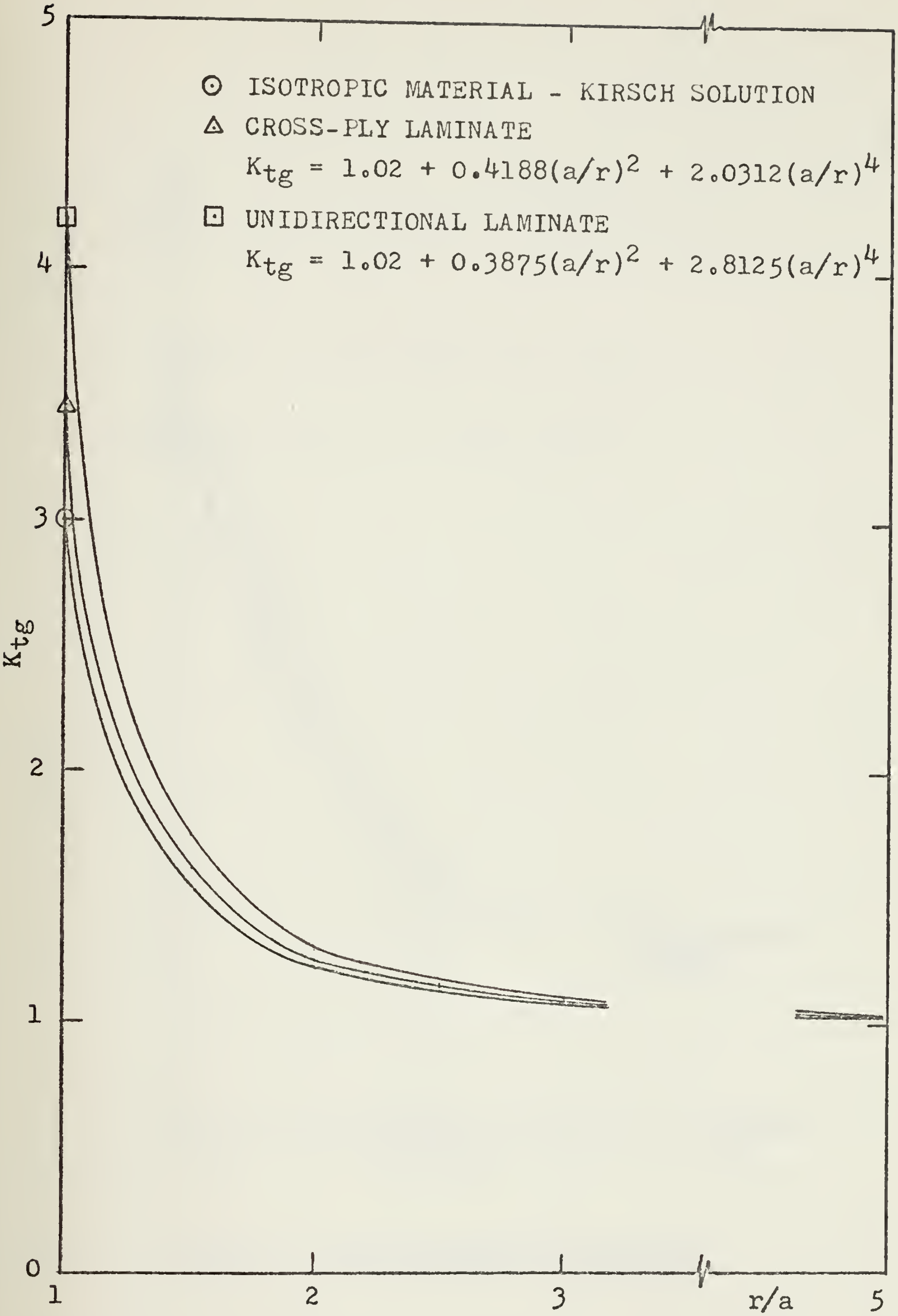

FIGURE 6. CURVES FOR APPROXIMATION OF THE STRESS CONCENTRATION FACTOR K_{tg} BASED ON KIRSCH SOLUTION FOR ISOTROPIC MATERIALS

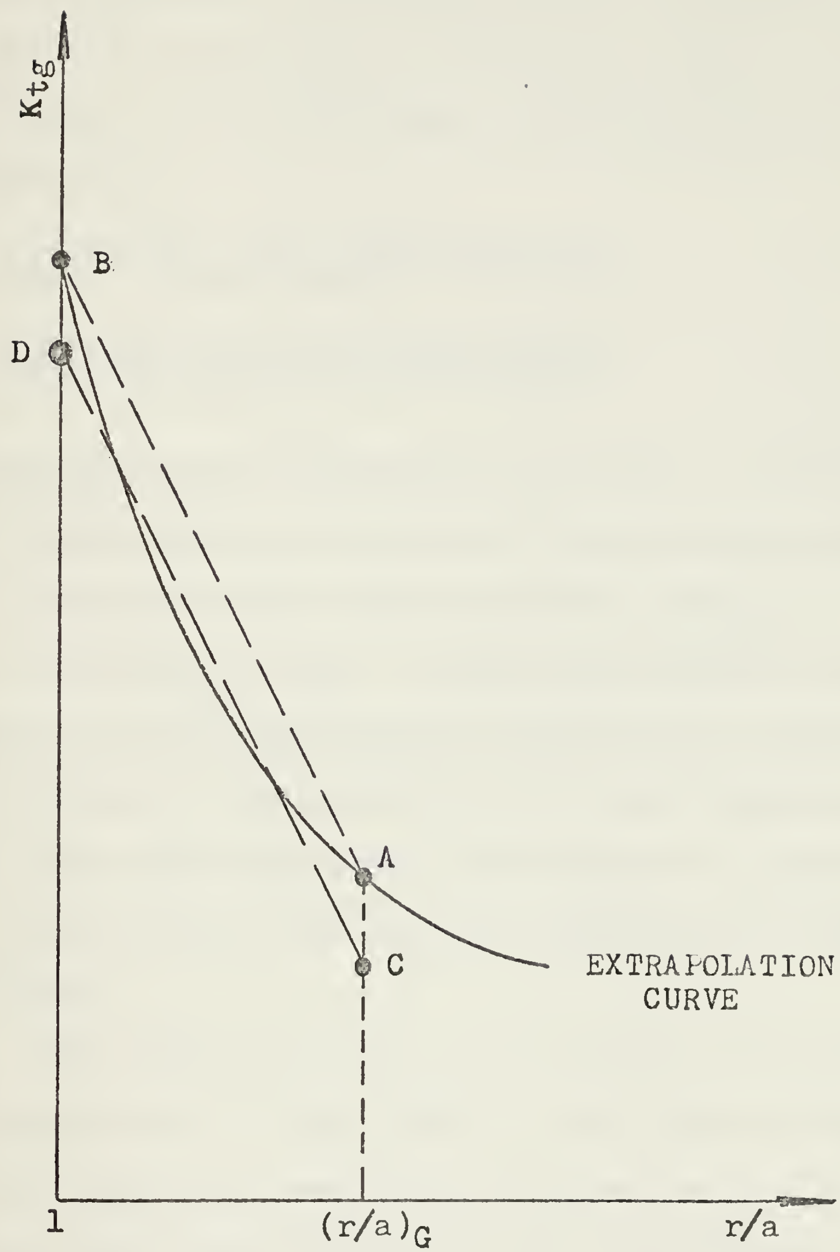

FIGURE 7. EXTRAPOLATION PROCEDURE

IX. CONCLUSIONS AND RECOMMENDATIONS

The conclusions and recommendations presented here, concerning the two types of fiberglass tested, the unidirectional and the cross-ply, can be logically divided into two parts:

1. Conclusions and recommendations based on the results,

2. Conclusions and recommendations based on the gained experience.

1. Conclusions and Recommendations Based on the Results

The goals that were expected to be met with the experimental program of this thesis were met, because:

a. As shown in Figs. 4 and 5 the variation of K_{tg} as a function of the dimensionless variable a/c was obtained. This variation is represented by the band between dashed lines. For both composites, the behavior is approximately the same as the one for isotropic materials [9] (shown in solid line).

b. The variation of K_{tg} as a function of the dimensionless variable e/c is very small. This result could be expected, since the same result is observed for isotropic materials [9], over the range of a/c investigated.

2. Conclusions and Recommendations Based on Gained Experience

During the early phases of the experiments a considerable fluctuation of the gage readings was observed and attributed to the heat transfer from the gages due to air currents in

the laboratory. It was necessary to isolate the specimen from the external air flow with a plastic skirt. In addition, shielding from the sun is advisable.

The size of the specimens used was a major source of scatter. It is recommended that future specimens have a size sufficient to provide a more uniform stress field away from the hole and the grip tabs.

Another source of error is the lack of uniformity of the thickness over the plate. In case it is necessary to test very thin plates, care should be taken to maintain uniform thickness within close tolerances.

The small thickness of the plates tested required installing the gages beside the holes. This required an extrapolation for K_{tg} from the gage position to the edge of the hole. Thicker specimens would have allowed installation of the gages on the inner surface of the hole. This is recommended.

It is also recommended that future investigators consider the grips as a possible source of error.

APPENDIX A. YOUNG'S MODULUS

The Young's Modulus for each of the materials used was obtained from solid rectangular specimens with a strain gage installed at the center. These specimens represent the three different types of laminates used in the test program, say: the unidirectional laminate used in the verification specimen, the unidirectional laminate used in specimens from IAI to IDIII, and the cross-ply laminate used in specimens IIAI to IIDIII. For all the cases mentioned above, the strain gage used was of the foil type.

The resultant Young's Modulus for each one of the laminates is, respectively: 6.31×10^6 psi $\pm$ 15 percent, 6.13×10^6 psi $\pm$ 12 percent, and 3.63×10^6 psi $\pm$ 10 percent, for the three types of laminates described above.

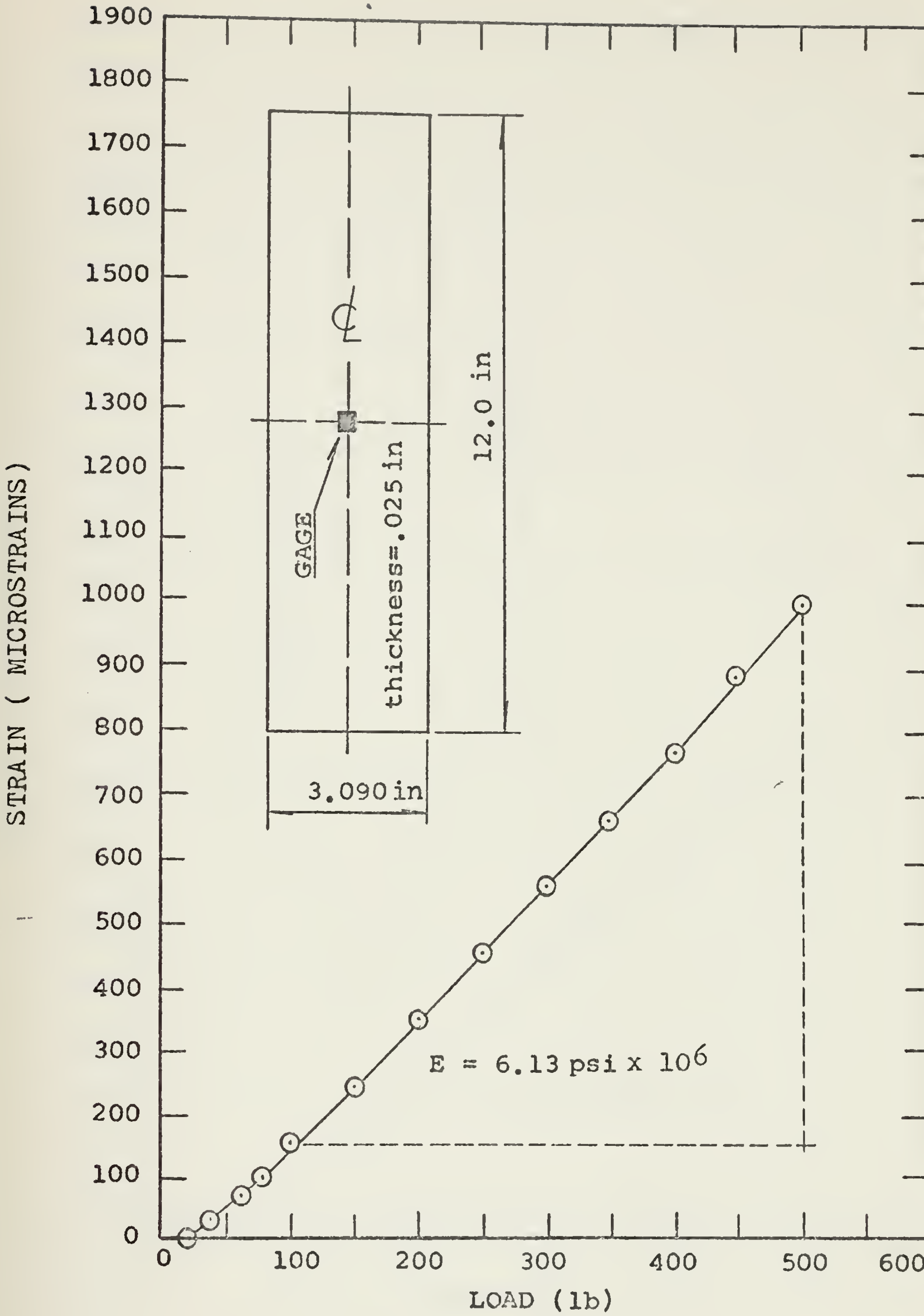

FIGURE 8. STRAIN AS A FUNCTION OF THE LOAD FOR UNIDIRECTIONAL LAMINATE

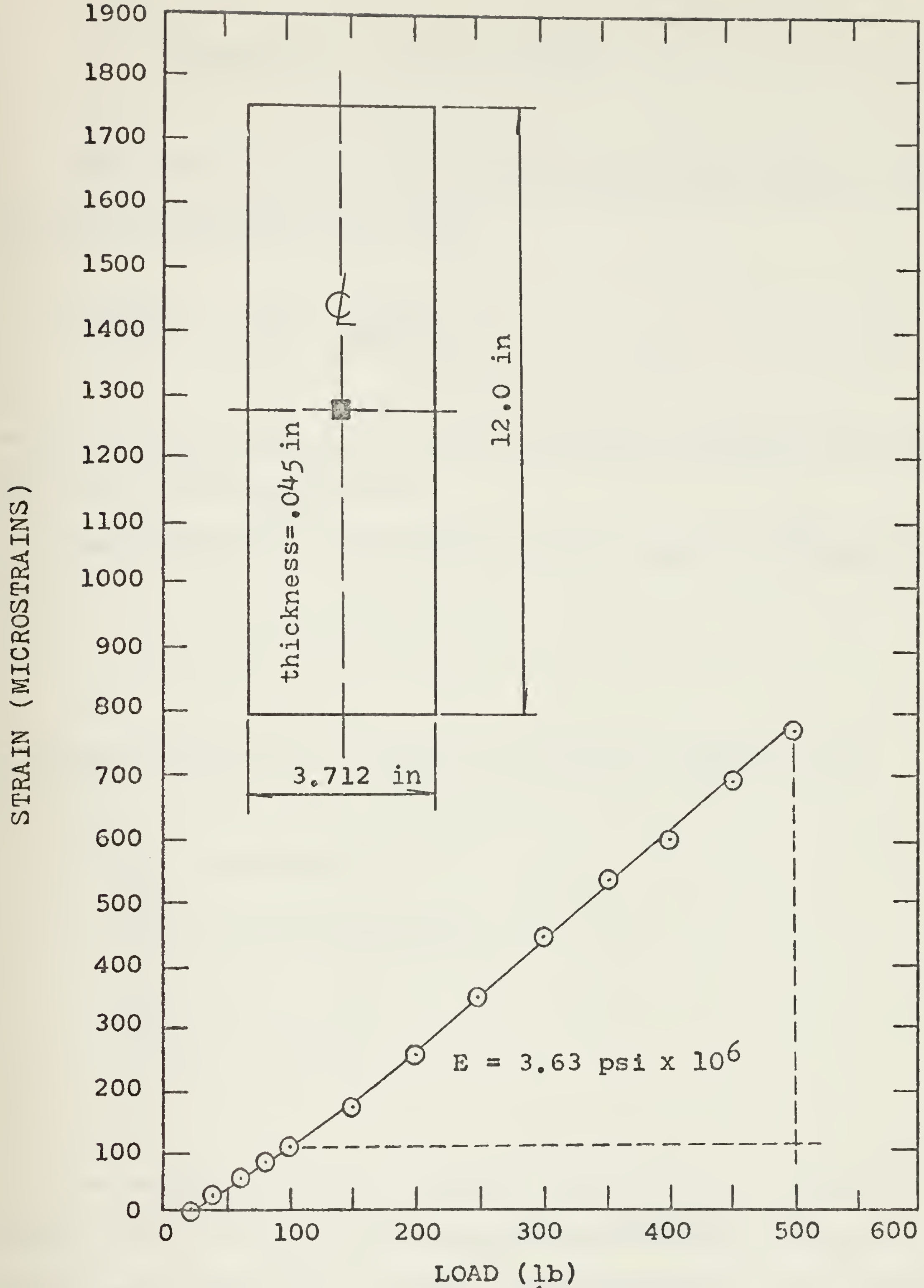

FIGURE 9. STRAIN AS A FUNCTION OF THE LOAD FOR CROSSPLY LAMINATE

APPENDIX B. ERROR ANALYSIS

The error analysis for Young's Modulus was based on the following equation [Ref. 24]:

$$\% \text{ ERROR} = \pm \frac{1}{\phi} \times \sum \left(\frac{\partial \phi}{\partial a} \omega_a\right)^2 \qquad (1)$$

where ϕ is the function, the a's are the variables, and the ω_a's are the uncertainties.

The formula for calculation of the Young's Modulus is:

$$E = S/\varepsilon = P/(b \times t \times \varepsilon)$$

The uncertainties involved in Young's Modulus are:

$\omega_P = \pm$ 20 lbf

$\omega_e = \pm$ 20 microstrains

$\omega_b = \pm$ 0.02 in

$\omega_t = \pm$ 0.002 in

The values of Young's Modulus obtained have relative errors of: 15, 12, and 10 percent respectively for the verification specimen laminate, unidirectional and cross-ply laminates.

The error analysis for the stress concentration factor K_{tg} was based on equation (1).

The formula for calculation of K_{tg} is:

$$K_{tg} = (E \times \varepsilon_H \times b \times t)/P$$

The uncertainties involved in K_{tg} are:

$$\omega_P = \pm\ 20\ \text{lbf}$$

$$\omega_\varepsilon = \pm\ 20\ \text{microstrains}$$

$$\omega_b = \pm\ 0.02\ \text{in}$$

$$\omega_t = \pm\ 0.002\ \text{in}$$

The values of K_{tg} obtained have relative errors of 21, 17, and 14 percent, respectively for verification specimen, unidirectional and cross-ply specimens.

APPENDIX C. ALUMINUM SPECIMEN TO CHECK THE VALIDITY OF THE EXPERIMENTS

After the end of the verification specimen tests, apparently all the sources of errors had been examined and corrected, but still a considerable scatter seemed to be present in the data. As a consequence, it was decided to create a special specimen made of known aluminum alloy, and to test it, following the same procedure as the one followed during the fiberglass specimens test. Overall information about the behavior of the tension test machine, the gages, and the strain gage indicator was obtained.

The aluminum specimen made of 2024 alloy is shown in Fig. 10.

From this test two conclusions were drawn: first, the stress concentration factor K_{tg} obtained from both gages (one inside the hole) did not present any appreciable tendency to vary with the load. These results are shown in Fig. 11. Secondly, good agreement between the experimental K_{tg} and the theoretical prediction was obtained.

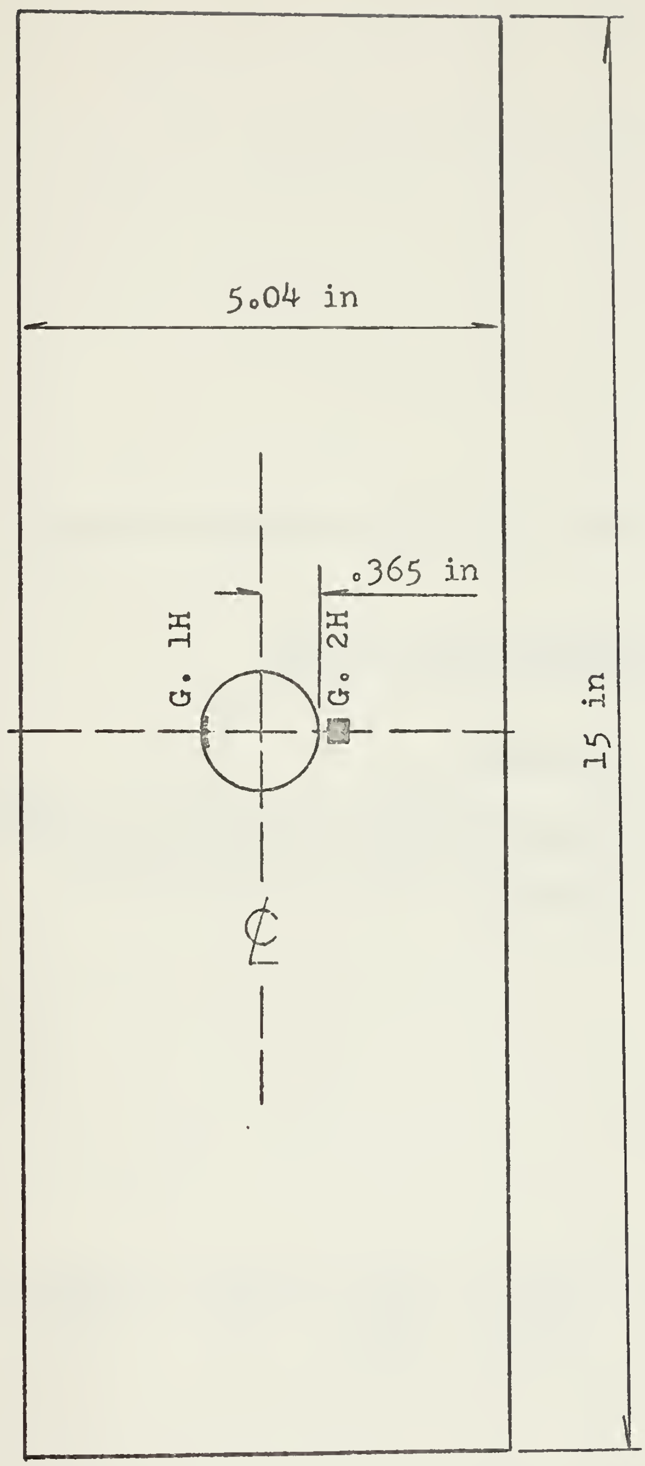

FIGURE 10. ALUMINUM SPECIMEN

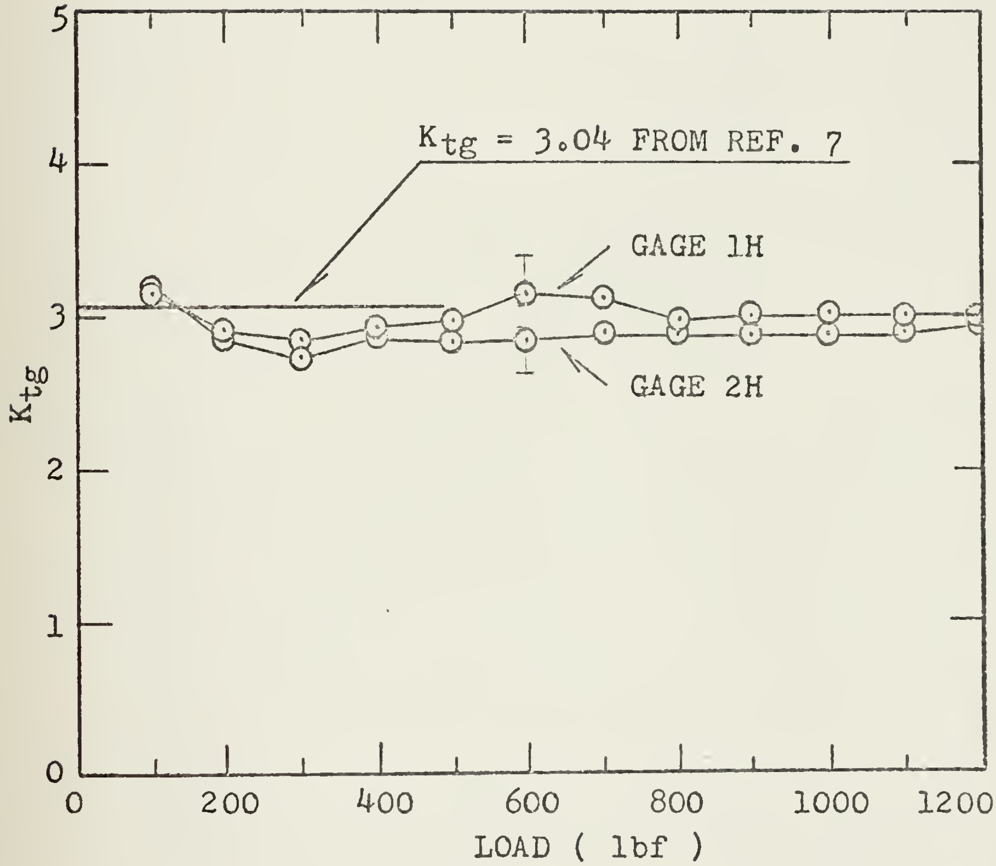

FIGURE 11. STRESS CONCENTRATION FACTOR K_{tg} AS A FUNCTION OF THE LOAD. ALUMINUM SPECIMEN

APPENDIX D. SAMPLE OF GAGE RESPONSE

In most of the cases the gages behavior were similar to the one shown in Fig. 12.

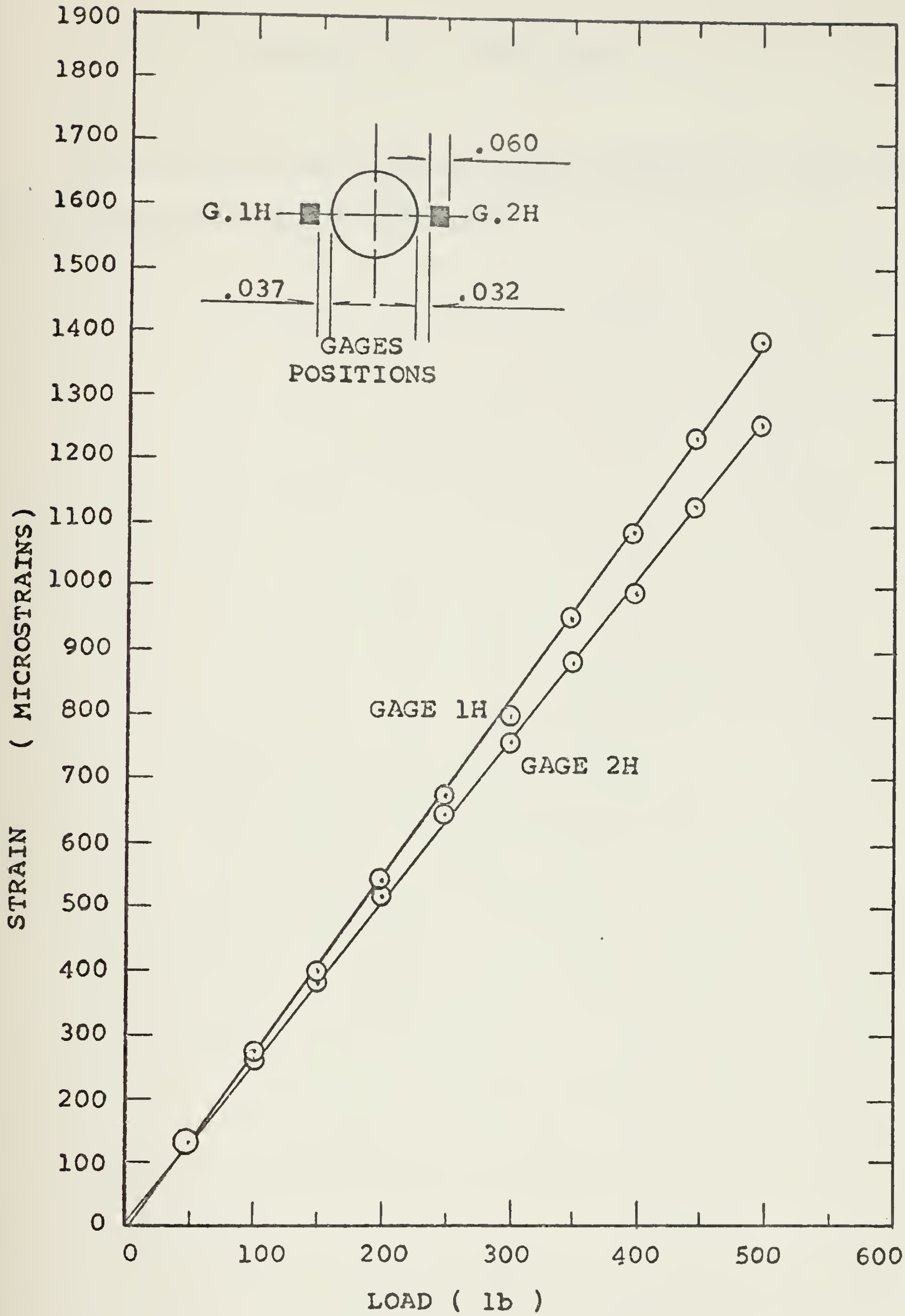

FIGURE 12. STRAIN AT SIDE OF THE HOLE AS A FUNCTION OF THE LOAD

APPENDIX E. TEST DATA

The data obtained from the gage readings is contained in the tables of this appendix.

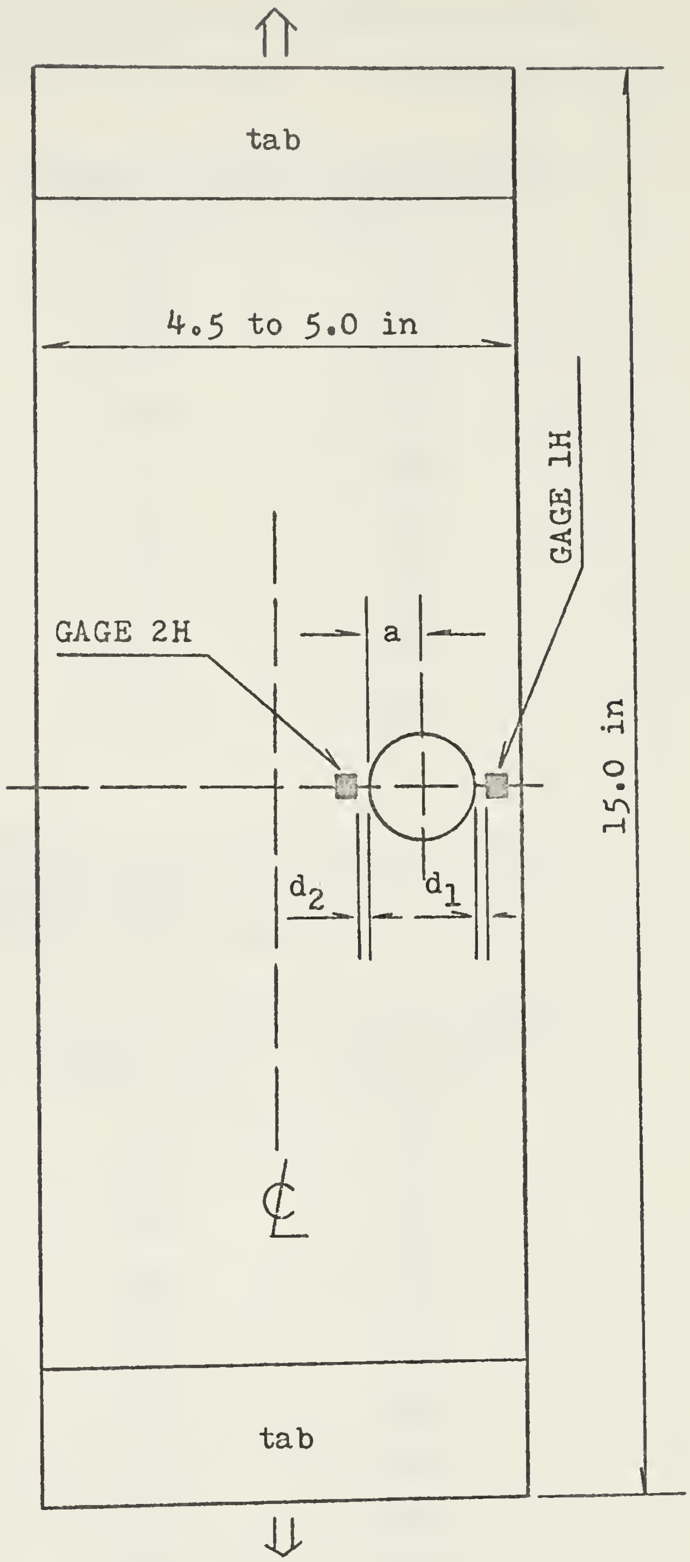

FIGURE 13. SPECIMEN DIMENSIONS AND STRAIN GAGE LOCATION PARAMETERS

TABLE 2. SPECIMEN IAI
STRAIN GAGE READINGS - GAGES 1H and 2H
AT SIDES OF HOLE
$d_1 = 0.031$, $d_2 = 0.031$, $a = 0.25$, $f = 1.5$

LOAD (lb)	AVERAGE STRAIN (MICROSTRAINS) GAGE 1H	GAGE 2H
50	65	75
100	160	162
150	250	252
200	376	370
250	502	485
300	625	605
350	750	725
400	880	840
450	1010	960
500	1150	1090

TABLE 3. SPECIMEN IAII
STRAIN GAGE READINGS - GAGES 1H and 2H
AT SIDES OF HOLE
$d_1 = 0.032$, $d_2 = 0.037$, $a = 0.25$, $f = 1.0$

LOAD (lb)	AVERAGE STRAIN (MICROSTRAINS) GAGE 1H	GAGE 2H
50	75	110
100	160	230
150	260	355
200	365	482
250	470	615
300	580	745
350	700	880
400	815	1010
450	935	1150
500	1060	1290

TABLE 4. SPECIMEN IAIII
STRAIN GAGE READINGS - GAGES 1H AND 2H
AT SIDES OF HOLE
$d_1 = 0.012$, $d_2 = 0.012$, $a = 0.25$, $f = 0.5$

LOAD (lb)	AVERAGE CORRECTED STRAIN (MICROSTRAINS) GAGE 1H	GAGE 2H
50	120	145
100	255	285
150	380	420
200	520	560
250	650	702
300	785	848
350	920	985
400	1060	1130
450	1195	1270
500	1340	1425

TABLE 5. SPECIMEN IBI
STRAIN GAGE READINGS - GAGES 1H AND 2H
AT SIDES OF HOLE
$d_1 = 0.037$, $d_2 = 0.037$, $a = 0.19$, $f = 1.5$

LOAD (lb)	AVERAGE CORRECTED STRAIN (MICROSTRAINS) GAGE 1H	GAGE 2H
50	100	75
100	230	172
150	342	282
200	460	400
250	565	515
300	675	631
350	780	741
400	860	860
450	975	979
500	1110	1110

TABLE 6. SPECIMEN IBII
STRAIN GAGE READINGS - GAGES 1H AND 2H
AT SIDES OF HOLE
$d_1 = 0.016$, $d_2 = 0.094$, $a = 0.19$, $f = 1.0$

LOAD (lb)	AVERAGE STRAIN (MICROSTRAINS) GAGE 1H	GAGE 2H
50	100	100
100	235	210
150	360	350
200	485	465
250	620	580
300	755	695
350	890	810
400	1025	925
450	1155	1050
500	1300	1180

TABLE 7. SPECIMEN IBIII
STRAIN GAGE READINGS - GAGES 1H AND 2H
AT DIES OF HOLE
$d_1 = 0.037$, $d_2 = 0.032$, $a = 0.19$, $f = 0.5$

LOAD (lb)	AVERAGE STRAIN (MICROSTRAINS) GAGE 1H	GAGE 2H
50	118	120
100	220	240
150	335	360
200	455	490
250	570	615
300	690	742
350	810	865
400	930	990
450	1050	1125
500	1170	1250

TABLE 8. SPECIMEN ICI
STRAIN DISTRIBUTION ALONG LINE THROUGH CENTER OF HOLE - STRAIN GAGE READINGS
$d_1 = 0.0$, $d_2 = 0.0$, $a = 0.13$, $f = 1.5$

LOAD (lb)	GAGES 1H	2H	3H	4H	5H	6H	7H	8H
50	107	76						
100	243	178	150	155	170	185	165	168
150	377	279						
200	520	381	300	318	338	350	310	325
250	663	488						
300	800	600	445	465	492	510	450	470
350	945	699						
400	1070	810	590	612	641	660	590	615
450	1220	912						
500	1360	1020	740	760	791	813	720	760

TABLE 9. SPECIMEN ICII
STRAIN GAGE READINGS - GAGES 1H AND 2H AT SIDES OF HOLE
$d_1 = 0.0$, $d_2 = 0.0$, $a = 0.13$, $f = 1.0$

LOAD (lb)	AVERAGE STRAIN (MICROSTRAINS) GAGE 1H	GAGE 2H
50	VERY LOW VALUES	150
100		290
150		430
200		568
250		711
300		844
350		971
400		1100
450		1250
500		1385

TABLE 10. SPECIMEN ICIII
STRAIN GAGE READINGS - GAGES 1H AND 2H
AT SIDES OF HOLE
$d_1 = 0.0$, $d_2 = 0.0$, $a = 0.13$, $f = 0.5$

LOAD (lb)	AVERAGE STRAIN (MICROSTRAINS) GAGE 1H	GAGE 2H
50	110	105
100	231	230
150	376	380
200	512	515
250	649	660
300	775	800
350	900	945
400	1035	1085
450	1155	1230
500	1300	1380

TABLE 11. SPECIMEN IIAI
STRAIN GAGE READINGS - GAGES 1H AND 2H
AT SIDES OF HOLE
$d_1 = 0.037$, $d_2 = 0.032$, $a = 0.25$, $f = 1.5$

LOAD (lb)	AVERAGE STRAIN (MICROSTRAINS) GAGE 1H	GAGE 2H
50	132	135
100	271	265
150	400	382
200	535	505
250	671	630
300	800	755
350	948	885
400	1090	1005
450	1230	1130
500	1380	1260

TABLE 12. SPECIMEN IIAII
STRAIN GAGE READINGS - GAGES 1H AND 2H
AT SIDES OF HOLE
$d_1 = 0.035$, $d_2 = 0.032$, $a = 0.25$, $f = 1.0$

LOAD (lb)	AVERAGE STRAIN (MICROSTRAINS) GAGE 1H	GAGE 2H
50	65	75
100	160	162
150	250	252
200	376	370
250	502	485
300	625	605
350	750	725
400	880	840
450	1010	960
500	1150	1090

TABLE 13. SPECIMEN IIAIII
STRAIN GAGE READINGS - GAGES 1H AND 2H
AT SIDES OF HOLE
$d_1 = 0.023$, $d_2 = 0.031$, $a = 0.25$, $f = 0.5$

LOAD (lb)	AVERAGE STRAIN (MICROSTRAINS) GAGE 1H	GAGE 2H
50	85	82
100	220	200
150	365	315
200	575	435
250	665	565
300	820	690
350	965	820
400	1120	940
450	1275	1065
500	1430	1200

TABLE 14. SPECIMEN IIBI
STRAIN GAGE READINGS - GAGES 1H AND 2H
AT SIDES OF HOLE
$d_1 = 0.032$, $d_2 = 0.030$, $a = 0.19$, $f = 1.5$

LOAD (lb)	AVERAGE STRAIN (MICROSTRAINS) GAGE 1H	GAGE 2H
50	135	159
100	272	295
150	400	415
200	528	543
250	650	670
300	765	800
350	885	920
400	1000	1045
450	1119	1165
500	1240	1290

TABLE 15. SPECIMEN IIBII
STRAIN GAGE READINGS - GAGES 1H AND 2H
AT SIDES OF HOLE
$d_1 = 0.031$, $d_2 = 0.031$, $a = 0.19$, $f = 1.0$

LOAD (lb)	AVERAGE STRAIN (MICROSTRAINS) GAGE 1H	GAGE 2H
50	105	125
100	230	251
150	345	372
200	470	500
250	585	620
300	700	745
350	815	860
400	940	985
450	1055	1110
500	1180	1240

TABLE 16. SPECIMEN IIBIII
STRAIN GAGE READINGS - GAGES 1H AND 2H
AT SIDES OF HOLE
$d_1 = 0.050$, $d_2 = 0.037$, $a = 0.19$, $f = 0.5$

LOAD (lb)	AVERAGE STRAIN (MICROSTRAINS) GAGE 1H	GAGE 2H
50	100	110
100	200	224
150	310	339
200	419	449
250	531	565
300	645	683
350	762	795
400	876	915
450	982	1030
500	1110	1139

TABLE 17. SPECIMEN IICI
STRAIN GAGE READINGS - GAGES 1H AND 2H
AT SIDES OF HOLE
$d_1 = 0.031$, $d_2 = 0.031$, $a = 0.13$, $f = 1.5$

LOAD (lb)	AVERAGE STRAIN (MICROSTRAINS) GAGE 1H	GAGE 2H
50	130	125
100	220	232
150	320	330
200	436	445
250	553	555
300	658	662
350	760	754
400	870	865
450	980	980
500	1082	1090

TABLE 18. SPECIMEN IICII
STRAIN GAGE READINGS - GAGES 1H AND 2H
AT SIDES OF HOLE
$d_1 = 0.0$, $d_2 = 0.0$, $a = 0.13$, $f = 1.0$

LOAD (lb)	AVERAGE STRAIN (MICROSTRAINS) GAGE 1H	GAGE 2H
50		
100	165	
150		
200	288	
250		
300	415	NO READINGS
350		
400	535	
450		
500	690	

TABLE 19. SPECIMEN IICIII
STRAIN GAGE READINGS - GAGES 1H AND 2H
AT SIDES OF HOLE
$d_1 = 0.0$, $d_2 = 0.0$, $a = 0.13$, $f = 0.5$

LOAD (lb)	AVERAGE STRAIN (MICROSTRAINS) GAGE 1H	GAGE 2H
50	170	165
100	265	242
150	355	320
200	450	400
250	540	480
300	630	560
400	810	715
450	905	800
500	1000	875

TABLE 20. SPECIMEN IDIII
STRAIN GAGE READINGS - GAGES 1H AND 2H
AT SIDES OF HOLE
$d_1 = 0.025$, $d_2 = 0.025$, $a = 0.31$, $f = 0.5$

LOAD (lb)			GAGES		
	1H	2H	3H	4H	5H
50	135	150	90	55	60
100	330	305	178	120	120
150	402	468	275	195	190
200	545	633	368	269	260
250	701	795	465	342	325
300	851	959	563	418	395
350	995	1120	660	496	466
400	1135	1290	759	572	540
450	1285	1452	860	652	615
500	1437	1630	965	740	690

TABLE 21. SPECIMEN IIDIII
STRAIN GAGE READINGS - GAGES 1H AND 2H
AT SIDES OF HOLE
$d_2 = 0.030$, $a = 0.31$, $f = 0.5$

LOAD (lb)		GAGES		
	2H	3H	4H	5H
50	120	90	70	65
100	235	190	140	140
150	418	290	225	230
200	520	388	310	310
250	661	490	390	390
300	800	580	470	470
350	937	675	550	550
400	1078	780	630	630
450	1225	875	705	710
500	1371	975	800	800

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